Folding Paper into Light: New Material, Form, and Making

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Abstract: In comparison to other fabrication techniques, folding or bending allows for complex and innovative structures formed with simple and low cost tools at the point of assembly. From flat sheet material, folded designs can be easily deployed into a three-dimensional volume and then can be collapsed back to a two-dimensional flat shape that is much smaller, for ease of shipping and storage. The goal of this paper is to systemically explore practical means of using mathematical and scientific origami in product design in order to seek innovative ways of form finding and making through the materiality of paper folding. Geometric tessellated patterns that can produce flat-foldable and rigid-foldable designs are focused, and variations of these crease patterns are explored to generate a variety of folded designs that can be used toward light sheds. A wide variety of durable conventional paper and technical paper materials that are appropriate for light sheds are compared and tested in terms of their material properties, functionalities and sustainability attributes. In order to investigate such issues, this paper focuses on a case study of a lighting brand, Folded Light Art. Currently technical production processes for cutting sheet materials, as well as traditional hand folding, are used to produce the scale paper models and 1:1 scale prototypes for the works of Folded Light Art. Luminary hardware is fabricated at local sheet metal shops. Potential CNC technologies of cutting, scoring and etching sheet materials are further explored. Though the current focus is on light sheds, this research can be applied to other folded designs that can potentially shift the paradigm in sustainable product design and development.

Keywords: Flat-foldability, Rigid-foldability, Mathematical Origami, Digital Cutting, Geometric Tessellation, Technical Paper, Folded Light Art

Introduction

aper is often thrown away or recycled after one-time or short-term use because of its fragility and commonality. Due to recent advances in the material science of paper development, paper has started to be used for technical components and for three dimensional products (Schmidt 2009). According to Schmidt, technical paper is a new type of material that is manufactured in a similar way to conventional paper and consists of similar raw materials. It is processed and shaped using the same technologies or processes as conventional paper and possesses qualities similar to conventional paper, such as its feel, weight, and format. Technical paper that is water- and weather-proof, tear-proof, and chemical resistant, is now being used for the manufacturing of light-weight and long lasting products. Product designers have begun to turn their attention to this cheap and common material and have started to recognize the sustainable value in it. For example, architects such as Frank Gehry and Shigeru Ban and renowned designer Issey Miyaki have used paper to produce their furniture, buildings, and even fashion. However, technical paper can do a lot more. For example, scientists have been experimenting with making graphene paper that is as strong as steel and electrically conductive (Chen et al. 2008, Ranjbartoreh et al. 2011). It is not too difficult to imagine a future in which the advance of technical paper can change the way we design a wide range of products. If so, what are the implications of the newly found materiality in technical paper for form generation and making in design?

Paper may be easily cut, scored, torn, rolled, or folded. Artist Paul Jackson has experimented with folding paper using only one single creases to transform a flat and passive sheet of paper into something that is structural, spatial and dynamic (Jackson 2011). One of the early design explorations through paper folding was found in Josef Albers' preliminary design class in the Bauhaus (Wingler and Stein 1969). When a flat piece of paper is folded, the stiffness of the paper is significantly increased. When paper is folded in certain tessellation patterns, the mechanical behavior of the paper is altered as it becomes deployable and kinetic. While paper folding is easy



to make, it is very difficult to model. Paper folding is a real physics. To model the paper folding in the computer, one needs to work in a simulated physical environment. Describing paper folding scientifically at a level of generalization and morphological representation requires complex mathematical modeling. Therefore, origami-inspired paper designs have been the subject of many scientific research projects. For example, mathematical theorems concerning geometric properties in folded paper have been studied (Demaine and O'Rourke 2008). Computer algorithms have been developed to help fold desirable objects and to understand the best ways to fold tray cartons (Mullineux 2010). Finite element analysis has been used to study the behavior of paper during the folding process (Beex and Peerlings 2009). Curved folding and creasing have been investigated. Many more research projects focus on understanding the computational complexity and geometric algorithms of paper folding and unfolding. While mathematical research into paper folding has been conducted in computer science, mathematics, engineering and material science, little such research has been conducted on paper folding in the field of design practice. This article is an attempt to fill this gap by systemically exploring practical means of using mathematical and scientific origami in product design in order to seek innovative ways of form finding and making through the materiality of paper folding in design.

Historically, paper folders have been concerned with the capability of morphogenesis and form self-generation from the properties of the material itself. In this article, paper folding is investigated as design artifacts: how they are made, what are their intrinsic properties, and how they can be contribute to the current design methodology. To experiment with form generation through paper folding, the easiest method is to work with the material tactilely, following a step by step procedure or algorithm: start with a flat sheet of paper, crease the paper according to a set of crease pattern, transform the paper by folding, turning, rotating, pulling, pushing, wrapping, etc., and end with a three-dimensional paper structure. The resulting three-dimensional paper structure can then be tested and scaled for the intended function. When paper folding is restricted to a set of constraints, it might seem difficult for form generation. However, these constraints might, at the same time, be easier for us to study and understand. In this article, only geometric tessellated crease patterns that can produce flat-foldable and rigid-foldable designs are focused upon and variations of these crease patterns are explored to generate a variety of folded designs that can create rigid three dimensional volume. A wide variety of durable conventional paper and technical paper materials that are appropriate for product design are compared and tested in terms of their material properties, functionalities, and sustainability attributes. In order to investigate such issues, we will examine a lighting brand, Folded Light Art. Currently technical production processes for cutting and etching sheet materials, such as digital cutting or laser cutting, as well as traditional hand folding, are used to make the scale paper models and 1:1 scale prototypes for the works of Folded Light Art. Luminary hardware is fabricated at local sheet metal shops. Potential CNC technologies of creasing and folding sheet materials are further explored in this article.

Deployable and Flat-foldable Origami Tessellations in Design

In comparison to other fabrication techniques, folding or bending allows for complex and innovative structures formed with simple and low cost tools at the point of assembly. From flat sheet material, folded designs can be easily deployed into a three-dimensional volume and then can be collapsed back to a two-dimensional flat shape that is much smaller for easier shipping and storage. Many folded designs are inspired by origami, the Japanese art of paper folding. The original purpose of origami is to create various shapes, ranging from animals figures to objects, both abstract and figurative, by folding a flat sheet of uncut paper. Constructing a three-dimensional structure from two-dimensional sheet material in origami has inspired designers and engineers to come up with novel ways to fabricate, assemble, store and morph structures that are

safe, efficient and energy saving (Edwin 2014), from collapsible medical stents for hearts (Kuribayashi et al. 2006) to airbags for cars.

Typically, geometric crease patterns are focused when paper folding is applied in design. The crease pattern refers to a set of mountain folded lines and valley folded lines appearing on a sheet of paper when a folded structure is open flat. Figure 1 shows an origami pinwheel figure (a) and the associated crease pattern (b). Solid lines and dash lines represent mountain folds and valley folds respectively.

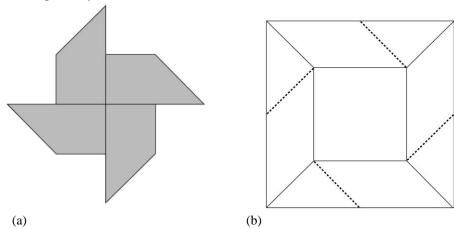


Figure 1: (a) An Origami Pinwheel; (b) Crease Pattern of (a). Source: Computer Simulation Using ORIPA (Mitani 2012)

Perhaps the interesting part of origami design is to fold a desired 2D or 3D shape from a single piece of paper. It has been mathematically proven that any 2D polygon or 3D polyhedral can be folded from a flat sheet of paper, as long as the paper is big enough (Demaine and O'Rourke 2008). Computer programs for origami or paper folding design have been explored by a few mathematicians and computer scientists, including Jun Mitani's ORIPA and Robert Lang's TreeMaker (Lang 2003, 1998). However, it has been quite difficult to come up with crease patterns that can generate desirable forms. While there are many different types of crease patterns in origami design, only origami tessellations employing crease patterns that are able to create self-deployable and flat-foldable structures are focused. Self-deplorability and flat foldability are two essential features when creating folded design that can be deployed into three dimensional structures and compressed back to two dimensional shapes.

One of the ways to understand deplorability in origami is that the folded three-dimensional forms are not locked and they can be folded into their final state by bending the material just at the crease lines. In a typical traditional origami model, such as the crane shown in Figure 2, folding and unfolding the model needs to follow a step by step sequence. In order to reach a folded state in a crane origami, one must finish a previous folded state. Therefore the folded structure can not be deployed simultaneously in a self-organized manner.

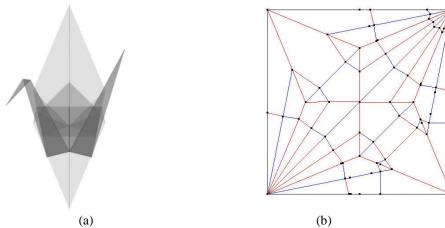


Figure 2: (a) An Origami Crane; (b) Crease Pattern of (a). Source: Computer Simulation Using ORIPA (Mitani 2012)

In contrast, in a deployable folded structure, the folding pattern needs to be self-organized. A folding pattern inspired by these sorts of self-organized creased patterns, such as the Miura pattern(Miura 2009), is the basis for a self-folding map and deployable solar cell array on Space Flyer Unit, a space platform launched in Japan in 1994-1995. Figure 3 shows various deployable states of a rigid-foldable design based on the Miura pattern, discovered by Japanese scientist Miura Ori when researching ways to uniformly buckle a thin plate structure. The primary deployable feature of Miura pattern includes its ability to deploy simultaneously in orthogonal directions and to retract and deploy on the same path ((Miura 2009).



Figure 3: Various Deployable States of a Miura Pattern.

Source: Computer Simulations Created Using FreeForm Origami (Tachi 2014)

While a folded structure is rigidly foldable, it is not necessary flat foldable. In order for a crease pattern to be flat foldable locally then a necessary and sufficient condition, called Kawasaki's Theorem, must be satisfied. Kawasaki's Theorem states that in order for a crease pattern to be flat foldable locally then the number of lines connecting to a single inner vertex (the points where crease lines meet) must be even and the sums of alternating angles must be 180 degrees. Figure 4 shows an example of a crease pattern with six lines connecting to a single vertex that satisfy the conditions listed in Kawasaki's Theorem. For an entire folded structure to be flat folded, the Kawasaki condition must be able to be applied to all the inner vertices of a crease pattern globally, and thus there will be no collision of the parts of the folded structure during assembly. Figure 5 shows an origami pinwheel design that is flat foldable and satisfies

Kawasaki's Theorem and an origami pinwheel design that doesn't satisfy Kawasaki's Theorem and therefore isn't flat foldable.

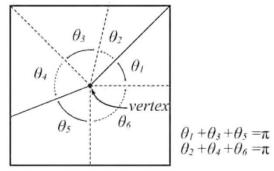


Figure 4. An example of a crease pattern with six lines connecting to a single vertex that satisfy the conditions listed in Kawasaki's Theorem (Mitani 2011)

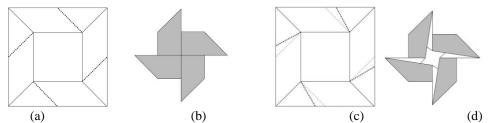


Figure 5. (a) Crease Pattern of a Flat-foldable Origami Pinwheel. (b) Flat-foldable Origami Pinwheel of (a). (c) Crease Pattern of a Non-flat-foldable Origami Pinwheel. (d) Non-flat-Foldable Origami Pinwheel of (c). (Source: Author)

In order to create large array of flat foldable and deployable design, only origami tessellations are focused. An origami tessellation is a folded design where both the crease pattern and the folded structure use continuous and repeated elements based on the symmetries found in 17 crystallographic design (Davis et al. 2013). One of the foremost pioneers of origami tessellation is Shuzo Fujimoto, who often uses motifs made from hexagons, squares and triangles (Rutzky and Palmer 2011). Beside catching the interests of origami artists, the origami tessellations, or the periodic folded forms, also caught the interests of computer scientists and artists such as Ren Resch (Resch 1968), David Huffman (Huffman 1976), and Paulo Taborda Barreto (Barreto 1997). Perhaps the most well know flat-foldable and deployable origami tessellations are Ron Resch's Waterbomb pattern, the Miura pattern, the Yoshimura pattern and Nojima's helical triangle pattern. Figure 6 shows these four aforementioned patterns and their folded forms in their respective deployable and flat-folded states.

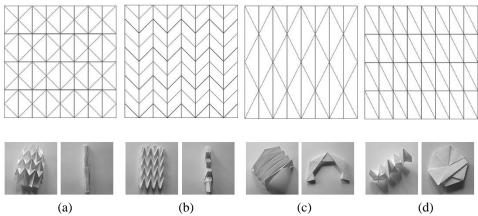


Figure 6. Crease Patterns and the Associate Folded Forms in Flat-foldable States and Deployable States. (a) Ron Resch's Waterbomb pattern. (b) Miura Pattern. (c) Yoshimura Pattern. (d) Nojima's Helical Triangle Pattern. (Source: Author)

Though it has been proven mathematically difficult to come up with original flat-foldable and self-organized crease patterns, original folding design that is based on the rearrangement of four aforementioned well-known patterns including the Ron Resch pattern (or Waterbomb pattern), the Miura pattern, the Yoshimura pattern and the Nojima Pattern (or Helical Triangle pattern), can be generated fairly easily. Several transformations of these four tessellated origami crease patterns are discussed below, while the designs that are able to generate self-supporting and self-enclosed structural volumes are focused. Because of the periodic symmetrical nature of these four patterns, the transformations that employ stretch, compression and shear, alone or in a combination, will result in the crease patterns that are symmetrical and that satisfy Kawasaki's Theorem. Figure 7 shows a crease pattern that is a simple compression of the Waterbomb pattern, producing a folded surface that is double-curved and that can be formed into an artistic torus. While periodic and flat-foldable tessellation can be generated by the aforementioned transformation, non-periodic transformations are also possible and they can produce unique deployable and flat-foldable structures. Figure 8 shows both periodic tessellations and nonperiodic tessellations, and the associated folded cylindrical structures that are deployable and flat-foldable by using compression, stretch, shear, as well as distortion of the original helical triangle crease patterns.

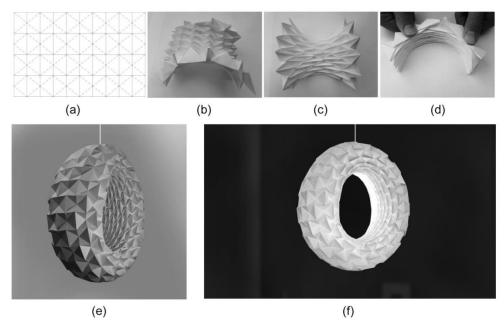


Figure 7. (a) A Compressed Waterbomb Pattern. (b) Folded Form in Deployable State. (c) Folded Surface Demonstrating Double Curvature Geometry. (d) Folded Form in Flat-Foldable State. (e) A Torus that is Folded from a Single Piece of Paper. (f) Lighted View of Torus.(Source: Author)

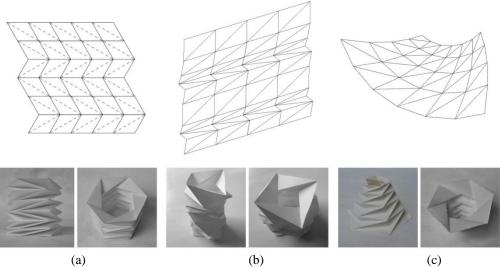


Figure 8. Transformation of Helical Triangular Pattern, Showing the Folded Deployable Structures and the Flat-folded Forms. (a) Symmetrically Sheared Transformation. (b) and (c) Asymmetrically Sheared Transformation. (Source: Author)

In many origami tessellations the base unit of a water-bomb pattern can be transformed into a Tree pattern with bounding polygon of valley folds, and an interior tree-like structure of mountain folds (Figure 9b). This Tree pattern can be characterized as the most basic repeating unit found in many tessellation designs (Davis et al. 2013). Any valley folds and mountain folds in the crease patterns can be replaced with these Tree patterns. Figure 9 shows the transformation

of the water-bomb pattern by Ron Resch by adding Tree patterns. By working with these most basic origami units, one can generate new tessellated crease patterns and folded designs.

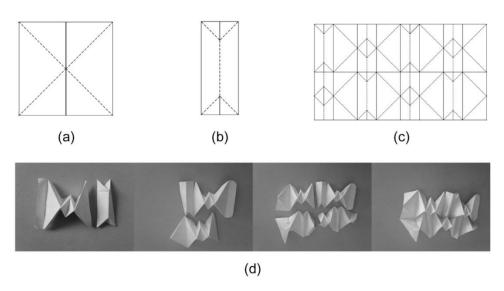


Figure 9. The Transformation of Ron Resch's Waterbomb Pattern. (a) A Basic Unit of Waterbomb Pattern. (b) A Basic Unit of Tree Pattern. (c) New Crease Pattern Combining Waterbomb Pattern and Tree Pattern. (d) Paper Models Showing the Transformation Process. (Source: Author)

The Materiality of Technical Paper

In art and design, the material, or the medium, plays an important role in the process of making. Though paper has been used for printing and for packaging, not until the past few years, due to advances in material science, has paper been used for constructing three-dimensional products that are aimed for long term usages. The material properties of paper are determined by its fiber, filler materials, binding agents and coating materials. Conventional paper is produced using wood-based pulp and chemical-based filler materials and binding agents, therefore it is fragile and can be easily broken down. When the paper is produced not from wood-based pulp but from raw materials such as carbon, plastic, bamboo, or glass, paper can be very strong and resistant to tear. Replacing conventional filler material such as chalk and titanium oxide with unusual filler materials such as ceramic particles, metal oxides, or silver ions, paper can have unique integrated functions. In addition, paper can be coated with silicone and other plastics, making it water-proof (Schmidt 2009). By adding innovative components and modifying manufacturing processes, technical paper can be processed and shaped as conventional paper, and so it possesses similar qualities, such as its feel, weight and format. For example, Graphene paper, a Nano-structured material processed from graphite, is six times lighter and ten times stronger in comparison to steel (Ranjbartoreh et al. 2011). For another example, photo-catalyst, or air-purifying paper, is a type of paper that is coated with photo-catalyst such as titanium oxide. When organic matter in the atmosphere comes into contact with titanium oxide, it is oxidized at an increased rate and breaks down into water and carbon dioxide. In Japan, photo-catalyst paper is used to purify the air in interior spaces in traditional sliding doors and lamps, and as wallpaper and curtains.

While technical paper has attracted a lot of attention from the art and design communities recently, the process of making tear-resistant paper and the concept of using paper other than in its daily common use are not new. For example, Washi paper (*Wa* means "Japanese" and *Shi* means "paper") has been used to make lamps and interior screens in Japan for centuries. Perhaps

one of the most well-known examples is the Washi paper lamps made by Akari Noguchi, a renowned mid-century modernist designer. Washi paper is made from the long inner fibers of three plants: kozo (mulberry tree), mitsumata, and gampi. Due to these raw materials and the traditional making techniques, Washi paper is highly workable and flexible, even when it is wet. In addition to its more common uses in the fine arts, its fabric like quality makes it suitable for applications in fashion, interior lighting and interior furnishing. Washi paper can also be laminated with plastic to add more durability, stiffness and water-resistance, and it is often used for interior and exterior sliding door screens, which are also called Shoji in Japan.

Table 1 shows a table of a variety of technical paper that have been tested for light sheds, including High Density Polyethylene (HDPE) sheet, Polyester paper, rag paper, Washi paper, and Elephant Hide paper. In addition to many aspects of physical properties, such as its texture, weigh, thickness, water-resistance, tensile strength and tear-resistance, light diffusion and permeation qualities are compared. Furthermore, special focuses are given to the paper memory and bending resistance to folds. Memory refers to how easy a flat fold can be made so that folded sheet lays flat. Bending resistance refers to the amount of force you need to apply to get a sharp crease and how strong the paper is while being curved(Garibi and Vishne 2015). The Memory and Bending Resistance are ranked from poor, fair, good, to great. Based on the comparison, HDPE sheet and technical Washi paper are among the best paper for folding and light shed applications because of their durability and light diffusion and permeation abilities. The economic HDPE sheet is made of 100 percent polymer that is bonded together by heat without any additive; therefore it is 100 percent recyclable. On the other hand, the more expensive technical Washi paper is made in a three-layer structure with polyester film as a core and Washi on both sides, therefore making it difficult to recycle.

Table 1	. Comparison of	a Variety of Tech	nical Papers	
E sheet	Polyester	Dog Donor	Technical	E
L sneet	Paper	Rag Paper	Washi Paper	İ

	HDPE sheet	Polyester Paper	Rag Paper	Technical Washi Paper	Elephant Hide Paper
Brand Name	Tyvek 1085D	DuroMeda DPM-5.7	Folio Legion Paper	High-tec Kozo	Zanders
Country of Origin	United States	United States	United States	Japan	Germany
Backlit Picture					
Texture	Vellum Textile	Smooth Plastic	Vellum Paper	Vellum Textile	Smooth Marble
Thickness	10.3 mil	5.7 mil	n/a	7.8 mil	n/a
Weight	108.8 gsm	n/a	250 gsm	n/a	110 gsm
Water Proof	No	yes	No	No	No
Water Resistant	Yes	yes	No	Yes	Yes
Tear Proof	No	Yes	No	Yes	No
Tear Resistant	Yes	Yes	No	Yes	Yes
Aging	Good	Great	Good	Great	Good
Memory	Good	Good	Great	Good	Great
Bending Resistance	Good	Great	Fair	Great	Fair
Fire Resistance	Great	Great	Poor	N/A	Poor
Light Diffusing ability	Great	Fair	Fair	Great	Good
Light Permeation	Great	Poor	Good	Great	Great for white color. Poor for dark color
Recyclability	Yes	No	Yes	No	Yes

Folded Light Art: A Case Study

Folded Light Art, a light shed brand exploring the tectonic relationship between forms, material, and making techniques through the art of origami, showcases a collection of deployable and flat foldable designs. The goal of Folded Light Art project is to create a self-enclosed volume to host a LED light source. Several paper models are folded and studied based on the alternations and combinations of the afore-mentioned patterns. The resulting volume can be of any regular polygonal shapes, such as a square, hexagon, or octagon. All of the light sheds in the Folded Light Art collection are flat foldable and deployable. Figure 10 shows a collection of Folded Light Art in various geometric volumes and the associated flat-folded packages.

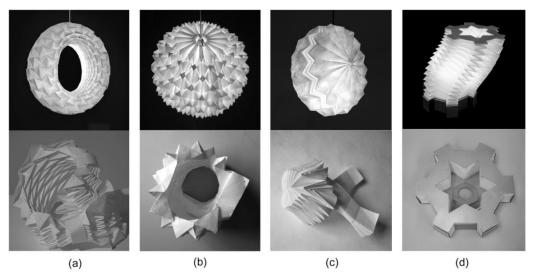


Figure 10. Examples from Folded Light Art Collection Showing the Light Shades and the Associated Flat Folded Packaging Ideas. (a) Torus. (b) Durian Durian. (c) Moonlight Mandala. (c) Booma Star Source: Author

For the light shades, durable spunbonded HDPE sheet is chosen for its durability, cost-effectiveness and recyclability. This polymer sheet is very versatile, offering a balance of physical characteristics that combine the best properties of paper, film, and cloth, and it can be cut and folded easily. It is made of a very fine and continuous polymer fiber that is first spun and then bonded together by heat and pressure, without additional chemicals and additives. Because of this, HDPE sheet is one hundred percent recyclable. It is super lightweight (lighter than paper), flexible, and resistant to water, abrasion, chemicals, and aging. Because of its polymer content, olefin has a great ability to diffuse light, while its thin fiber under lighting conditions provides additional visual interest, thus making it an ideal material for light shade.

Many of the light sheds in Folded Light Art collection were folded from one single piece of HDPE sheet. While it is a very durable material and it is tough to tear, HDPE is very easy to cut and score. These patterns were then hand creased and folded into the desirable three-dimensional forms. A Graphtec 6000-120 was used to perforate and cut line patterns on the HDPE sheet. Graphtec 6000-120 is a cost-effective digital tool for cutting large format vinyl with a self-sticking backing layer in a roll. In order to cut and perforate a standard sheet of HDPE sheet using the Graphtec 6000-120, a rather cumbersome step is added by first applying repositionable adhesive to a backing layer and then attaching this layer as a backing material to the HDPE sheet. Other high-end digital cutting tools, such as the Zund digital cutting system, can also be appropriate for cutting, perforating and creasing HDPE sheets. The Zund digital cutting system is one of the high-end industry standards for high-volume digital cutting in various industries including graphics, packaging, leather, textile and composite. Tools that come with the Zund digital cutting system are able to use various cutting angles to create complex 3D structural designs from foamcore, sandwich boards and other composite materials.

Custom stainless steel luminary hardware that works with the light sheds were fabricated at local sheet metal fabricators (Figure 11). While LED lights produce significantly less heat in comparison to the conventional light bulbs, careful design decisions are considered to keep the light source a few inches from the light sheds. Though there is not sign of heat on the light sheds after the lights have been on for at least eight hours, further testing needed to be conducted by an UL (Underwriters Laboratories) consulting and testing company.



Figure 12. Custom Made Luminary Hardware. (a) Luminary Hardware for Folded Light Art Pendant Lamp Collection.

(b) Luminary Hardware for Folded Light Art Table Lamp Collection.

Source: Author

Conclusions

This article has described an innovative design methodology that places material and the tactile qualities associated with form finding and making at the center of a design process. Using paper as the primary material in product design requires a different approach than the conventional CAD centered design. In paper folding the form generation is the direct result of material manipulation through a series of actions by hands. While current CAD technologies are very efficient at producing precise drawings for industrial production, the automations are often inadequate for design that is based on a tactile design process. Therefore, it is imperative for the design community to participate at and collaborate with computer scientists, engineers and mathematicians in order to create innovative computer programs to simulate the intrinsic behaviors and properties found in sheet material folding, such as paper folding. Furthermore, developing and using new sustainable material, such as technical paper, requires innovative and open-minded approaches from designers, researchers, marketing professionals and product developers. Cutting and folding the design products, such as light sheds, described in this article to industrial standards for mass production, however, requires new technologies, such as robots for making complex folds with paper materials. Though the current focus is on light sheds, this research can be applied to other folded designs that can potentially shift the paradigm in design by materiality, form finding, and making. Further research and design explorations need to be conducted in order to fully understand the potential of technical paper for the functional design of other objects, and in architecture, as well.

REFERENCES:

- Barreto, Paulo Taborda. 1997. "Lines Meeting on a Surface: The 'Mars' Paperfolding." proceedings at the *Second International Meeting of Origmai Science and Scientific Origami*. at Seian University of Art and Design, Japan.
- Beex, L. A. A., and R.H. Peerlings. 2009. "An experimental and computational study of laminated paperboard creasing and folding." *International Journal of Solids and Structures* no. 46:4192-4207.
- Chen, H, M Muller, K Gilmore, G Wallace, and D Li. 2008. "Mechanically Strong, Electrically Conductive, and Biocompatible Graphene Paper." *Advanced Materials* no. 20:3557–3561.

- Davis, Eli, Erik D. Demaine, Martin L. Demaine, and Jennifer Ramseyer. 2013. "Reconstructing David Huffman's Origami Tessellations." *Journal of Mechanical Design* no. 135 (11):111010-1–111010-7.
- Demaine, Erik D., and Joseph O'Rourke. 2008. *Geometric folding algorithms : linkages, origami, polyhedra*. Cambridge; New York: Cambridge University Press.
- Edwin, A; Peraza-Hernandez; et al 2014. "Origami-inspired active structures: a synthesis and review." *Smart Material and Structure* (23).
- Garibi, Ilan, and Gabi Vishne. 2015. *Paper Reviews* 2015 [cited January 20 2015]. Available from http://www.happyfolding.com/paper-reviews_introduction.
- Huffman, D.A. 1976. "Curvature and creases: A primer on paper." *IEEE Transactions on Computers* no. C-25 (10):1010-1019.
- Jackson, Paul. 2011. Folding Techniques for Designers: from sheet to form. London: Laurence King Pub.
- Kuribayashi, K., K. Tsuchiya, Z. You, D. Tomus, M. Umemoto, T. Ito, and M. Sasaki. 2006. "Self-Deployable Origami Stent Grafts as a Biomedical Application of Ni-rich TiNi Shape Membory Alloy Foil." *Mater, Sci. and Engr.* no. A419:131-137.
- Lang, Robert J. 2015. *TreeMaker* 1998 [accessed January 30 2015]. Available from http://www.langorigami.com/science/computational/treemaker/treemaker.php.
- Lang, Robert J. 2003. Origami design secrets: mathematical methods for an ancient art. Natick, MA: A.K. Peters.
- Miura, K. 2009. The Science of Miura-Ori: A Review. In *Fourth International Meeting of Origami: Science, Mathematics, and Education*, edited by Robert Lang: A K Peters, Ltd. Natick, Massachusetts.
- Mullineux, G.; Mathews, J. 2010. "Constraint-based simulation of carton folding operations." Compter-Aided Design no. 42:257-265.
- Mitani, J 2011. A "Method for Designing Crease Patterns for Flat-Foldable Origami with Numerical Optimization." *Journal for Geometry and Graphics* 15 (2): 195–201
- Mitani, J. 2012. ORIPA: Origami Pattern Editor 2012 [accessed June 3 2015].. Available from http://mitani.cs.tsukuba.ac.jp/oripa/.
- Ranjbartoreh, A, Bei Wang, Xiaoping Shen, and Guoxiu Wang. 2011. "Advanced mechanical properties of graphene paper" *Journal of Applied Physics* no. 109 (1).
- Resch, Ronald D. 1968. Self- Supporting Structural Unit Having a Series of Repetitious Geometric Modules. edited by US Patent.
- Rutzky, Jeffrey, and Chris K. Palmer. 2011. *Shadowfolds: surprisingly easy-to-make geometric designs in fabric*. New York: Kodansha International.
- Schmidt, Petra. 2009. Unfolded: paper in design, art and industry. Boston, MA: Birkhauser.
- Tachi, Tomohiro 2013. Freeform Origami 2014 [accessed June 12, 2013]. Available from http://www.tsg.ne.jp/TT/software/.
- Wingler, Hans Maria, and Joseph Stein. 1969. *The Bauhaus: Weimar, Dessau, Berlin, Chicago*. Cambridge, Mass.,: MIT Press.

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